SPONNE SCHOOL MATHEMATICS DEPARTMENT

In this booklet you will find 7 sections. Each section covers a topic from your Higher GCSE course that will be encountered again in your Year 12 course.

<u>This booklet forms part of your compulsory summer task. You should complete this work on paper, showing all working out. You should mark your work using the answers at the end of the booklet. The other part of your compulsory task is the reseach presentation.</u>

You will also have a test in the first few weeks back in September covering a range of topics, including those covered in this booklet and some that you will be taught in your year 12 lessons.

Contents

Linear Equations

An equation that only includes variables with a power of 1 is a *linear equation*. All linear equations will have graphs that form a straight line.

Example 1:

Solve $\frac{5x-2}{7} = 9$

× ×

$5x - 2 = 63$

+ +

$5x = 65$

+ +

$x = 13$

> To help decide the order: imagine secret brackets around the numerator of the fraction.
> $\frac{(5x-2)}{7}$

Minimum methods acceptable: As e.g. with no arrows

Example 2:

Solve $2x + 4 = 7x - 11$

\- -

$4 = 5x - 11$

+ +

$15 = 5x$

÷ ÷

$3 = x$

> Collect x terms first. Move the SMALLEST amount of x's to the biggest

Minimum methods acceptable: As e.g. with no arrows

Exercise 1:

1) Solve these equations involving divisions. Write your solution as a fraction when required.

a) $\frac{x}{2} - 5 = 4$ $\qquad$ $\frac{x+3}{4} = 12$

b) $3 + \frac{x}{4} = 10$ $\qquad$ c) $\frac{5x}{6} = \frac{1}{4}$

2) Solve these equations involving unknowns on both sides. Write your solution as a fraction when required.

a) $5m + 6 = 3m + 12$ $\qquad$ c) $5q - 4 = 3 - q$

b) $2p + 4 = p - 3$ $\qquad$ d) $7 - 3x = 5 - 2x$

3) Solve these equations involving brackets. Write your solution as a fraction when required.

a) $e + 3(e + 1) = 2e$
b) $5(f + 6) = 35f$
c) $3(2g + 1) + 2(g - 1) = 23$
d) $5h - 3(h - 1) = 39$

4) Solve these more complex equations. Write your solution as a fraction when required.

a) $\frac{21}{x} = 7$
b) $30 = \frac{6}{y}$
c) $\frac{2x-1}{3} = \frac{x}{2}$
d) $\frac{12}{2x-3} = 4$
e) $\frac{6}{x} - 3 = 7$
f) $\frac{5}{x+5} = \frac{15}{x+7}$

Quadratic Equations

An equation that only includes variables with a power of 2 is a *quadratic equation*. All quadratic equations will have graphs that form ∪ or ∩ shaped parabola.

Factorising Quadratic Expressions:

Example 1:

a) Factorise $x^2 + 5x - 24$

$x^2 + 5x - 24$

Product = -24 Sum = +5

-4 × 6 = 2 ×

-6 × 4 = -2 ×

-3 × 8 = +5 ✓

Solution: $(x - 3)(x + 8)$

b) Factorise $x^2 - 49$

$x^2 + 0x - 49$

Product = -49 Sum = +0

-7 × 7 = 0 ✓

Solution: $(x - 7)(x + 7)$

c) Factorise $6x^2 - x - 2$

$6x^2 - 1x - 2$

6 × -2

Product = -12 Sum = -1

-4 × 3 = -1 ✓

Separate x: $6x^2 - 4x + 3x - 2$

Factorise pairs: $2x(3x - 2) + 1(3x - 2)$

Solution: $(2x + 1)(3x - 2)$

Example 2:

Hence solve a) $x^2 + 5x - 24 = 0$

$\therefore (x - 3)(x + 8) = 0$

So $x - 3 = 0$ or $x + 8 = 0$

$x = 3$ or $x = -8$

b) $x^2 - 49 = 0$

$\therefore (x - 7)(x + 7) = 0$

So $x - 7 = 0$ or $x + 7 = 0$

$x = 7$ or $x = -7$

c) $6x^2 - x - 2 = 0$

$\therefore (2x + 1)(3x - 2) = 0$

So $2x + 1 = 0$ or $3x - 2 = 0$

$x = -\frac{1}{2}$ or $x = \frac{2}{3}$

Completing the Square:

Example 3: Write in the form $(x + a)^2 + b = 0$ and hence solve $x^2 + 6x - 16 = 0$

Compare $(x + a)^2 = x^2 + 2ax + a^2$ with the 1st two terms of the equation $x^2 + 6x$

Remove extra a^2 term

Include 3rd term of original equation

$\therefore\ 2a = 6$

$a = 3$ Substitute into required form: $(x + 3)^2 - 3^2 - 16 = 0$

Simplify: $(x + 3)^2 - 25 = 0$ (Completed Square Form)

Rearrange to solve: $(x + 3)^2 = 25$

$(x + 3) = \pm 5$

$x = -3 \pm 5 \qquad \therefore\ x = 2$ or $x = -8$

Using the Quadratic Formula:

REMEMBER
A quadratic equation MUST equal ZERO before you solve it.

Example 4:

Use the quadratic formula to solve $3x^2 - 7x + 2 = 0$

Substitute values from $ax^2 + bx + c = 0$ into $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\therefore\ x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times 3 \times 2}}{2 \times 3} = \frac{7 \pm \sqrt{49 - 24}}{6} = \frac{7 \pm \sqrt{25}}{6} = \frac{7 \pm 5}{6} = \frac{1}{3} \text{ or } 2$$

Exercise 2:

1) Factorise the following quadratic expressions:

a) $x^2 - 5x + 4$
b) $x^2 + 7x + 10$
c) $x^2 - 2x - 15$
d) $5y + 6 + y^2$
e) $10 - 11x + x^2$
f) $y^2 - y - 12$
g) $x^2 - 121$
h) $4x^2 - 81$
i) $4x^2 - 5x - 6$
j) $2x^2 - 5x + 3$
k) $15x^2 + 31x + 10$
l) $6x^2 - x - 1$

2) Solve the following equations by first factorising them:

a) $y^2 - 3y = 0$
b) $y^2 - 3y - 4 = 0$
c) $6x^2 - 11x - 7 = 0$
d) $x^2 + 2x = 35$
e) $3y^2 + 5y = 2$

3) Solve the following equations by first completing the square, give answers to 1 dp:

a) $x^2 + 10x + 3 = 0$
b) $x^2 - 8x - 2 = 0$
c) $x^2 + 3x - 4 = 0$

4) Solve the following equations using the quadratic formula, give answers to 1 dp:

a) $x^2 - 4x + 1 = 0$
b) $x^2 - 5x + 1 = 0$
c) $4x^2 + 9x + 1 = 0$
d) $4x^2 - 2x = 3$
e) $1 = x^2 - 8x + 2$

Simultaneous Equations

Problems that involve more than one equation and more than one unknown that are to be solved at the same time with the same values are known as *"Simultaneous Equations"*.

Linear Simultaneous Equations, Elimination Method:

Example 1:

Solve the following pair of equations simultaneously:

① $3x + 2y = 29 \rightarrow \times 4$
② $4x - 3y = -1 \rightarrow \times 3$

The aim is to get the same amount of one unknown by multiplying one or both equations by a constant

③ $12x + 8y = 116$ −
④ $12x - 9y = -3$

Unknowns have the SAME SIGN → SUBTRACT
DIFFERENT SIGNS → ADD

$17y = 119$
$y = 7$

Substitute into ① to find x: $3x + 14 = 29$
$x = 5$

Check results in ② $4 \times 5 - 3 \times 7 = 20 - 21 = -1$ ✓

Linear Simultaneous Equations, Substitution Method:

Example 2:

Solve the following pair of equations simultaneously:

① $3x + y = 22$
② $5x + 2y = 40$

The aim is to get one of the unknowns on its own and then substitute it into the other equation

① $y = 22 - 3x \rightarrow$ ② $5x + 2(22 - 3x) = 40$
$5x + 44 - 6x = 40$
$x = 4$

Substitute into ① to find y: $y = 22 - 3 \times 4 = 10$

Quadratic Simultaneous Equations:

Example 3:

Solve the following equations simultaneously:

① $y = 2x + 2$
② $y = x^2 - 1$

The aim is to substitute the linear equation into the quadratic equation.

See Factorising and Solving. Page 4

$2x + 2 = x^2 - 1$
$0 = x^2 - 2x - 3$
$0 = (x - 3)(x + 1)$
$x = 3$ or $x = -1$

Substitute into ① to find y:
$x = 3, y = 8$ $x = -1, y = 0$

Exercise 3:

1) Solve the following simultaneous equations using the elimination method:

a) $2x + 5y = 24$
$4x + 3y = 20$

b) $2a + 3b = 9$
$4a + b = 13$

c) $x - 2y = -4$
$3x + y = 9$

d) $5x - 7y = 27$
$3x - 4y = 16$

2) Solve the following simultaneous equations using the substitution method:

a) $x + 3y = 5$
$2x + y = 5$

b) $x - y = 2$
$3x + y = 10$

c) $a + 4b = 6$
$8b - a = -3$

d) $2x = 4 + z$
$6x - 5z = 18$

3) Solve the following linear and quadratic simultaneous equations:

a) $y = x^2 - 2x$
$y = x + 4$

b) $y = 7x - 8$
$y = x^2 - x + 7$

c) $y = x^2 - 3x + 7$
$5x - y = 8$

d) $y = 9x - 4$
$y = 2x^2$

Inequalities

Equations involving greater than >, less than <, greater than or equal to $\geq$ or less than or equal to $\leq$ are called *"Inequalities"*. Inequalities have a set of solutions.

Linear Inequalities:

REMEMBER
Change the direction of the inequality when you × or ÷ by a negative number.

Example 1:

Solve the following inequalities:

a) $3x + 7 < 31$
$3x < 24$
$x < 8$

b) $8 - 5x \geq 68$
$-5x \geq 60$
$x \leq -12$

See Solving Linear Equations. Page 2

Quadratic Inequalities:

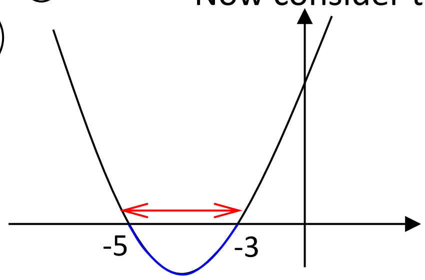
Example 2:

Solve the following inequality: $x^2 + 8x + 15 < 0$

$(x + 3)(x + 5) < 0$ so $x^2 + 8x + 15 = 0$ when $x = -3$ or $x = -5$

Now consider the graph of $y = x^2 + 8x + 15$

See Solving Quadratic equations. Page 4



So the graph has $y < 0$ when x is between -5 and -3

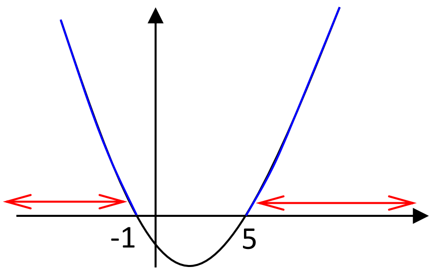
Solution: $-5 < x < -3$

Example 3:

Solve the following inequality: $x^2 - 4x - 5 > 0$

$(x - 5)(x + 1) > 0$ so $x^2 - 4x - 5 = 0$ when $x = -1$ or $x = 5$

Now consider the graph of $y = x^2 - 4x - 5$



So the graph has $y > 0$ when x is less than -1 or greater than 5

Solution: $x < -1$ or $x > 5$

(Ensure you write the solutions separately in this case)

Exercise 4:

1) Solve the following inequalities:

   a) $x - 3 > 10$
   b) $x + 1 < 0$
   c) $2x + 1 \leq 6$
   d) $5x < x + 1$
   e) $3x + 1 < 2x + 5$
   f) $2(x + 1) \geq x - 7$
   g) $3(x - 1) < 2(1 - x)$
   h) $4 - 2x \leq 2$

2) Solve the following quadratic inequalities:

   a) $x^2 + 7x + 12 < 0$
   b) $x^2 - 8x - 9 > 0$
   c) $x^2 - 144 \leq 0$
   d) $12x^2 - 16x + 5 < 0$
   e) $4x^2 - 3x - 10 > 0$
   f) $x^2 - 14x + 49 \leq 0$

Algebraic Fractions

Any fraction that involves an unknown is an *"Algebraic Fraction"*. You may be asked to *simplify* expressions or *solve* equations involving algebraic fractions.

Simplifying Algebraic Fractions, Adding & Subtracting:

Example 1:

Simplify the following algebraic fractions:

REMEMBER
Before you can add or subtract fractions you must have the same denominators.

a) $\frac{x}{6}+\frac{x}{8}$

$$=\frac{4x}{24}+\frac{3x}{24}=\frac{7x}{24}$$

REMEMBER
Only add the numerators.

b) $\frac{3}{x-2}+\frac{5}{x+3}$

$$=\frac{3(x+3)}{(x-2)(x+3)}+\frac{5(x-2)}{(x-2)(x+3)}$$

$$=\frac{3x+9}{(x-2)(x+3)}+\frac{5x-10}{(x-2)(x+3)}$$

$$=\frac{8x-1}{(x-2)(x+3)}$$

c) $\frac{12}{x+5}-\frac{2}{x+1}$

$$=\frac{12(x+1)}{(x+5)(x+1)}-\frac{2(x+5)}{(x+1)(x+5)}$$

$$=\frac{12x+12}{(x+5)(x+1)}-\frac{2x+10}{(x+1)(x+5)}$$

$$=\frac{12x+12-(2x+10)}{(x+5)(x+1)}$$

$$=\frac{10x+2}{(x+5)(x+1)}$$

Simplifying Algebraic Fractions, Multiplying & Dividing:

Example 2:

Simplify the following algebraic fractions:

REMEMBER
Never Divide: Flip the second fraction over & multiply.

REMEMBER
Always factorise before you multiply or divide.

a) $\frac{x}{x^2+5x+6}\times\frac{x^2+3x}{x+1}$

$$=\frac{x}{(x+2)\cancel{(x+3)}}\times\frac{x\cancel{(x+3)}}{x+1}$$

$$=\frac{x^2}{(x+2)(x+1)}$$

b) $\frac{x(x+2)}{x^2+8x-9}\div\frac{x^2+5x}{x-1}$

$$=\frac{\cancel{x}(x+2)}{(x+9)\cancel{(x-1)}}\times\frac{\cancel{x-1}}{\cancel{x}(x+5)}$$

$$=\frac{(x+2)}{(x+9)(x+5)}$$

REMEMBER
Cancel whole brackets from any numerator & denominator.

Solving Equations involving Algebraic Fractions:

👁 Example 3:

Solve the following equations:

a)
$$\frac{x}{x^2+5x+4}\times\frac{x^2+6x+8}{x+2}=3$$
$$\frac{x}{(x+1)\cancel{(x+4)}}\times\frac{\cancel{(x+2)}\cancel{(x+4)}}{\cancel{(x+2)}}=3$$
$$\frac{x}{(x+1)}=3$$
$$x=3(x+1)$$
$$x=3x+3$$
$$-2x=3$$
$$x=-\frac{3}{2}=-1\cdot5$$

When you have a single fraction on one side multiply up to remove all fractions.

b)
$$\frac{3}{x+1}+\frac{5}{x+2}=7$$
$$\frac{3(x+2)}{(x+1)(x+2)}+\frac{5(x+1)}{(x+1)(x+2)}=7$$
$$\frac{8x+11}{(x+1)(x+2)}=7$$
$$8x+11=7(x+1)(x+2)$$
$$8x+11=7x^2+21x+14$$
$$7x^2+13x+3=0 \text{ (using quadratic formula)}$$
x = -0.27 or x = -1.59

Exercise 5:

1) Simplify the following algebraic fractions:

a) $\frac{7a^2b}{35ab^2}$ b) $\frac{5ab}{15a+10a^2}$ c) $\frac{18a-3ab}{6a^2}$

d) $\frac{4ab+8a^2}{2ab}$ e) $\frac{x^2+2x}{x^2-3x}$ f) $\frac{x^2-3x}{x^2-2x-3}$

g) $\frac{x^2+6x+5}{x^2-x-2}$ h) $\frac{x^2-4x-21}{x^2-5x-14}$ i) $\frac{x^2+7x+10}{x^2-4}$

2) Write the following expressions as a single fraction:

a) $\frac{x-1}{3}+\frac{x+2}{4}$ b) $\frac{x-3}{3}-\frac{x-2}{5}$

c) $\frac{3}{4x}+\frac{2}{5x}$ d) $\frac{3}{4x}-\frac{2}{3x}$

e) $\frac{3}{x-2}+\frac{4}{x}$ f) $\frac{2}{x+3}-\frac{5}{x-1}$

3) Write the following expressions as a single fraction:

a) $\frac{x^2-3x-40}{x^2+2x}\times\frac{x^2+5x+6}{x^2-25}$ b) $\frac{x^2}{x^2+2x}\div\frac{x}{x+2}$

4) Solve the following equations giving answers to two decimal places where necessary:

a) $\frac{2}{x}+\frac{2}{x+1}=3$ b) $\frac{3}{x-1}+\frac{3}{x+1}=4$ c) $\frac{2}{x-2}+\frac{4}{x+1}=3$

Indices

An *"Index"* is also known as a power. The plural of index is *"Indices"*.

Index

Base $\rightarrow$ $\mathbf{2}^{3}$

Simplifying Indices:

You can only simplify indices when the bases are the same.

The Rules:

- $3^2 \times 3^5 = \underbrace{3 \times 3}_{3^2} \times \underbrace{3 \times 3 \times 3 \times 3 \times 3}_{3^5} = 3^7 = 3^{2+5}$ $\qquad y^a \times y^b = y^{a+b}$
- $5^6 \div 5^4 = \dfrac{5 \times 5 \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}\ 1} = \dfrac{5^2}{1} = 5^2 = 5^{6-4}$ $\qquad y^a \div y^b = y^{a-b}$
- $(3^2)^5 = 3^2 \times 3^2 \times 3^2 \times 3^2 \times 3^2 = 3^{2+2+2+2+2} = 3^{10} = 3^{2\times 5}$ $\qquad (y^a)^b = y^{a\times b}$
- $4^0 = 1$ $\qquad y^0 = 1$
- $9^{½} = \sqrt{9} = 3, \quad 8^{⅔} = (\sqrt[3]{8})^2 = 2^2 = 4$ $\qquad y^{n/m} = (\sqrt[m]{y})^n$
- $5^{-3} = \dfrac{1}{5^3} = \dfrac{1}{125}$ $\qquad y^{-n} = \dfrac{1}{y^n}$

Example 1:

Simplify the following expressions:

a) $c^4 \times c^7 = c^{4+7} = c^{11}$

b) $p^4 \div p^{-6} = p^{4-(-6)} = p^{10}$

c) $(r^4)^6 = r^{4\times 6} = r^{24}$

Example 2:

Evaluate the following:

a) $4^{-½} = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2}$

b) $(6^{½})^3 \times 6^{½} = 6^{3/2} \times 6^{½} = 6^{3/2 + ½} = 6^2 = 36$

c) $49^{3/2} = (\sqrt{49})^3 = 7^3 = 343$

d) $2.25^{-½} = \left(\dfrac{9}{4}\right)^{-1/2} = \left(\dfrac{4}{9}\right)^{1/2} = \dfrac{\sqrt{4}}{\sqrt{9}} = \dfrac{2}{3}$

Exercise 6:

1) Simplify the following expressions:

a) $x^3 \times x^4$

b) $m^3 \div m^2$

c) $y^{½} \times y^{½}$

d) $w^{-7} \times w^2$

e) $(k^{½})^6$

f) $(x^{-3})^{-2}$

g) $2x^2 \times 3x^2$

h) $(2x)^2 \times (3x)^3$

2) Evaluate the following quantities:

a) $100^{3/2}$

b) $(5^{-4})^{½}$

c) $81^{¼} \div 16^{¼}$

d) $0{\cdot}01^{½}$

e) $0{\cdot}04^{½}$

f) $\left(3\frac{3}{8}\right)^{1/3}$

g) $\left(11\frac{1}{9}\right)^{-1/2}$

h) $\left(\frac{1}{8}\right)^{-2}$

i) $\left(\frac{9}{25}\right)^{-1/2}$

Surds

Surds are irrational numbers and are a more accurate way of writing answers.

Simplifying surds:

Example 1:

a) $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

$b)\ 5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

$= 5\sqrt{6} - 2\sqrt{4}\sqrt{6} + \sqrt{49}\sqrt{6}$

$= 5\sqrt{6} - 4\sqrt{6} + 7\sqrt{6} = 8\sqrt{6}$

ALL PARTS MUST HAVE THE SAME SURD IN THEM TO BE ABLE TO ADD OR SUBTRACT.

Rationalising surds:

👁 Example 2:

$$a)\ \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{3}$$

Change the sign so that the terms containing surds in the expansion cancel out.

$$b)\ \frac{1}{3+\sqrt{2}} = \frac{1}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$$

$$= \frac{3-\sqrt{2}}{(3+\sqrt{2})(3-\sqrt{2})} = \frac{3-\sqrt{2}}{9-3\sqrt{2}+3\sqrt{2}-2}$$

$$= \frac{3-\sqrt{2}}{7}$$

Exercise 7:

1. Simplify the following surds

(i)	$\sqrt{8}$	(ii)	$\sqrt{50}$	(iii)	$\sqrt{48}$
(iv)	$\sqrt{216}$	(v)	$\sqrt{63}$	(vi)	$\sqrt{300}$
(vii)	$\sqrt{6}\times\sqrt{27}$	(viii)	$\sqrt{12}\times\sqrt{15}$	(ix)	$\sqrt{10}\times\sqrt{24}\times\sqrt{15}$

2. Multiply out the brackets and simplify where possible:

(i)	$\left(1+\sqrt{2}\right)\left(3-\sqrt{2}\right)$	(ii)	$\left(2-\sqrt{3}\right)\left(3+2\sqrt{3}\right)$
(iii)	$\left(3-2\sqrt{5}\right)\left(1-3\sqrt{5}\right)$	(iv)	$\left(\sqrt{2}+2\sqrt{3}\right)\left(5\sqrt{2}-\sqrt{3}\right)$
(v)	$\left(\sqrt{7}+\sqrt{2}\right)\left(\sqrt{7}-\sqrt{2}\right)$	(vi)	$\left(3-\sqrt{2}\right)^2$

3. Rationalise the denominators of the following surds:

(i)	$\frac{3}{\sqrt{3}}$	(ii)	$\frac{1}{\sqrt{5}}$	(iii)	$\frac{1+\sqrt{2}}{\sqrt{2}}$
(iv)	$\frac{1}{\sqrt{3}+1}$	(v)	$\frac{\sqrt{2}}{2-\sqrt{2}}$	(vi)	$\frac{1-\sqrt{3}}{2-\sqrt{3}}$
(vii)	$\frac{1+2\sqrt{5}}{3-\sqrt{5}}$	(viii)	$\frac{1+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$	(ix)	$\frac{\sqrt{6}+\sqrt{3}}{\sqrt{6}-\sqrt{3}}$

Answers

Exercise 1:

1)
a) $x = 18$
b) $x = 28$
c) $x = 45$
d) $x = \frac{3}{10}$

2)
a) $m = 3$
b) $p = -7$
c) $q = 1\frac{1}{6}$
d) $x = 2$

3)
a) $e = -1\frac{1}{2}$
b) $f = 1$
c) $g = 2\frac{3}{4}$
d) $h = 18$

4)
a) $x = 3$
b) $y = \frac{1}{5}$
c) $x = 2$
d) $x = 3$
e) $x = \frac{3}{5}$
f) $x = -4$

Exercise 2:

1)
a) $(x-1)(x-4)$
b) $(x+5)(x+2)$
c) $(x-5)(x+3)$
d) $(x+2)(x+3)$
e) $(x-1)(x-10)$
f) $(x+3)(x-4)$
g) $(x-11)(x+11)$
h) $(2x-9)(2x+9)$

i) $(4x+3)(x-2)$
j) $(2x-1)(x-1)$
k) $(5x+2)(3x+5)$
l) $(3x+1)(2x-1)$

2)
a) $y(y-3) = 0$
$y = 0$ or $y = 3$
b) $(y-4)(y+1) = 0$
$y = 4$ or $y = -1$
c) $(2x+1)(3x-7) = 0$
$x = -\frac{1}{2}$ or $x = 2\frac{1}{3}$
d) $(x+7)(x-5) = 0$
$x = -7$ or $x = 5$
e) $(3y-1)(y+2) = 0$
$y = \frac{1}{3}$ or $y = -2$

3)
a) $(x+5)^2 - 22 = 0$
$x = -0{\cdot}3$ or $x = -9{\cdot}7$
b) $(x-4)^2 - 18 = 0$
$x = -0{\cdot}2$ or $x = 8{\cdot}2$
c) $(x + \frac{3}{2})^2 - \frac{25}{4} = 0$
$x = 1$ or $x = -4$

4)
a) $x = 3{\cdot}7$ or $x = 0{\cdot}3$
b) $x = 4{\cdot}8$ or $x = 0{\cdot}2$
c) $x = -0{\cdot}1$ or $x = -2{\cdot}1$
d) $x = 1{\cdot}2$ or $x = -0{\cdot}7$
e) $x = 7{\cdot}9$ or $x = 0{\cdot}1$

Exercise 3:

1)
a) $x = 2, y = 4$
b) $a = 3, b = 1$
c) $x = 2, y = 3$
d) $x = 4, y = -1$

2)
a) $x = 2, y = 1$
b) $x = 3, y = 1$
c) $a = 5, b = \frac{1}{4}$
d) $x = \frac{1}{2}, z = -3$

3)
a) $x = 4, y = 8$
$x = -1, y = 3$
b) $x = 3, y = 13$
$x = 5, y = 27$
c) $x = 3, y = 7$
$x = 5, y = 17$
d) $x = \frac{1}{2}, y = \frac{1}{2}$
$x = 4, y = 32$

Exercise 4:

1)

a) $x > 13$
b) $x < -1$
c) $x \leq 2{\cdot}5$
d) $x < \frac{1}{4}$
e) $x < 4$
f) $x \geq -9$
g) $x < 1$
h) $x \geq 1$

2)

a) $-4 < x < -3$
b) $x < -1$, $x > 9$
c) $-12 \leq x \leq 12$
d) $\frac{1}{2} < x < \frac{5}{6}$
e) $x < -1\frac{1}{4}$, $x > 2$
f) $x = 7$

Exercise 5:

1)

a) $\frac{a}{5b}$
b) $\frac{b}{3+2a}$
c) $\frac{6-b}{2a}$
d) $\frac{2(b+2a)}{b}$
e) $\frac{x+2}{x-3}$
f) $\frac{x}{x+1}$
g) $\frac{x+5}{x-2}$
h) $\frac{x+3}{x+2}$
i) $\frac{x+5}{x-2}$

2)

a) $\frac{7x+2}{12}$
b) $\frac{2x-9}{15}$
c) $\frac{23}{20x}$
d) $\frac{1}{12x}$
e) $\frac{7x-8}{x(x-2)}$
f) $\frac{-3x-17}{(x+3)(x-1)}$

3)

a) $\frac{(x-8)(x+3)}{x(x-5)}$
b) 1

4)

a) $x = -\frac{2}{3}$ or $x = 1$
b) $x = -\frac{1}{2}$ or $x = 2$
c) $x = 0$ or $x = 3$

Exercise 6:

1)

a) x^7
b) m
c) y
d) w^{-5}
e) k^3
f) x^6
g) $6x^4$
h) $108x^5$

2)

a) 1000
b) $\frac{1}{25}$
c) 1·5
d) $\frac{1}{10}$
e) $\frac{1}{5}$
f) 1·5
g) $\frac{3}{10}$
h) 64
i) $1\frac{2}{3}$

Exercise 7:

1) $i)\ 2\sqrt{2}$
$ii)\ 5\sqrt{2}$
$iii)\ 4\sqrt{3}$
$iv)\ 6\sqrt{6}$
$v)\ 3\sqrt{7}$
$vi)\ 10\sqrt{3}$
$vii)\ 9\sqrt{2}$
$viii)\ 6\sqrt{5}$
$ix)\ 60$

2)
$i)\ 1 + 2\sqrt{2}$
$ii)\ \sqrt{3}$
$iii)\ 33 - 11\sqrt{5}$
$iv)\ 4 + 9\sqrt{6}$
$v)\ 5$
$vi)\ 11 - 6\sqrt{2}$

3) $i)\ \sqrt{3}$
$ii)\ \frac{\sqrt{5}}{5}$
$iii)\ \frac{\sqrt{2}+2}{2}$
$iv)\ \frac{\sqrt{3}-1}{2}$
$v)\ \sqrt{2}+1$
$vi)\ -1-\sqrt{3}$
$vii)\ \frac{13+7\sqrt{5}}{4}$
$viii)\ \sqrt{3}-\sqrt{2}+\sqrt{6}-2$
$ix)\ \frac{9+2\sqrt{18}}{3}$