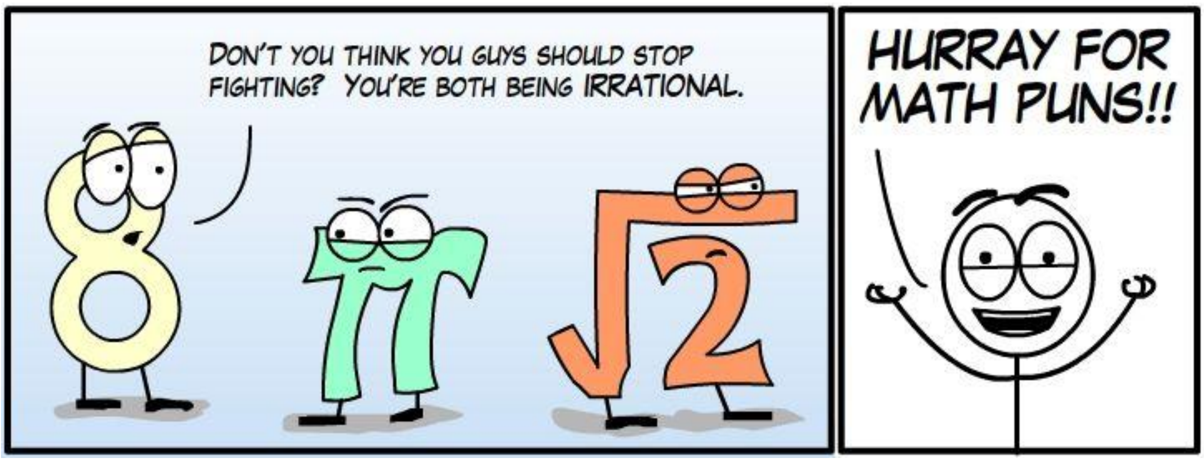
FURTHER MATHEMATICS

INTO THE SIXTH FORM

TRANSITION TASK

Please work through the booklet answering any questions that have been set. Research anything you do not know. Complete all sections with full written working.

ENJOY!!



Different Number Systems

Research the following Number Systems. Write down a sentence describing each system and then write down the symbol used for each one.

Integers

Natural Numbers

Rational Numbers

Irrational Numbers

Real Numbers

Complex Numbers

Number Systems

Write down two examples of each of the following

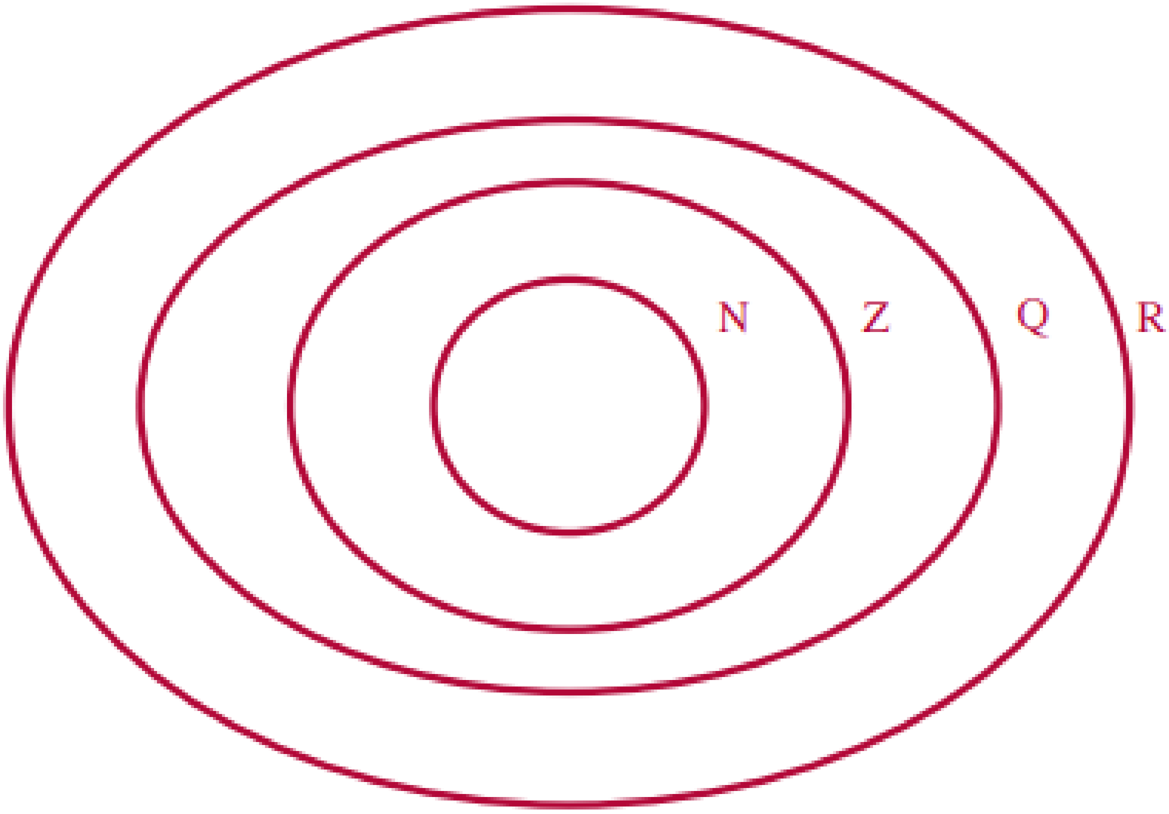
Natural numbers	
Positive Integers	
Negative Integers	
Integers	
Rational numbers which are positive and not natural numbers	
Rational numbers which are also integers	
Irrational numbers	
Real numbers	
Real numbers which are natural numbers, integers and rational numbers	
Complex numbers	

More Number Systems

Place the following numbers in their correct positions in the Venn Diagram below.

$$5, \quad \sqrt{5}, \quad \frac{15}{3}, \quad -\frac{2}{3}, \quad 0.3333333\ldots\ldots,$$

$$0.27272727\ldots\ldots\ldots., \quad \sqrt{4}, \quad \pi, \quad -3$$



<u>Complex Numbers</u>

Complex numbers involve both real and imaginary numbers. We can extend the number system to include the number i, this is called an imaginary number and has the properties:

$$i = \sqrt{-1} \qquad\qquad i^2 = -1$$

Using these facts, simplify:

1. i^5

2. i^9

3. i^{33}

4. $i^{16} + i^{10} + i^8 - i^{14}$

5. $i^{12} \times 3i^2 \times 2i^8$

6. $4i^3 + 7i^9$

7. $13i^8 - 4i^{14}$

8. $(3i^5)^2$

Complex Numbers

A complex number is made up of a real part and an imaginary part. In the complex number $3 + 7i$ the 3 is the real part and the 7 is the imaginary part.

Fill in the real and imaginary parts of each complex number.

Complex Number	Real part	Imaginary part
3		
0		
$2 + 7i$		
$4 - 3i$		
$-5 + 7i$		
$\frac{2}{3} + 5i$		
$2i$		
i		
$7 - \frac{4}{11}i$		
$5 + 6i$		
9		
$-\frac{1}{2} - \frac{2}{3}i$		

Adding and Subtracting Complex Numbers

To add or subtract complex numbers collect the real parts together and then collect the imaginary parts.

e.g. $(2 + 3i) + (4 - i) = (2 + 4) + (3 - 1)i = 6 + 2i$

$(7 - 4i) - (5 - 2i) = (7 - 5) + (-4 + 2)i = 2 - 2i$

Now you do:

1	$(12 + 4i) + (7 - 11i)$	
2	$(7 - 2i) + (9 - 4i)$	
3	$(4 - 6i) + (-5 - i)$	
4	$(3 - 8i) - (2 - 4i)$	
5	$(-12 - 5i) - (-2 - 8i)$	
6	$\left(2 + \frac{1}{3}i\right) + \left(3 - \frac{5}{6}i\right)$	
7	$\left(4 + \sqrt{-16}\right) + \left(-5 - \sqrt{-25}\right)$	
8	$z_1 = 5 + I$, $z_2 = -4 + 6i$ $z_3 = -11 + 2i$ Calculate $(z_1 + z_2) - z_3$	
9	$\left(4 - \sqrt{-50}\right) - \left(3 + \sqrt{-8}\right)$	
10	$z_1 = a + bi$ $z_2 = c + di$ $z_1 + z_2 =$ $z_2 + z_1 =$ $z_1 - z_2 =$ $z_2 - z_1 =$	

Multiplying Complex Numbers

Multiplying complex numbers is the same as expanding out brackets, remembering that $i^2 = -1$.

Now multiply out the following and simplify as much as possible.

1	$3(2 + 4i)$	
2	$(5 + 3i)i$	
3	$(2 - 7i)(3 + 4j)$	
4	$(4 - i)(3 + 2i)$	
5	$(2 + 3i)(2 - 3i)$	
6	$(2 + 5i)^2$	

Dividing with Complex numbers

To divide by a complex number you can use a method similar to that used to rationalise a fraction with a surd in the denominator.

$$\frac{7 + 4i}{3 - 2i} = \frac{(7 + 4i)(3 + 2i)}{(3 - 2i)(3 + 2i)}$$
$$= \frac{21 + 14i + 12i - 8}{9 + 4}$$
$$= \frac{13 + 26i}{13}$$
$$= 1 + 2i$$

$(3 + 2i)$ and $(3 - 2i)$ are called conjugates, or complex conjugates.

Now simplify the following, leaving numbers as fractions where needed.

1	$\frac{1}{i}$	
2	$\frac{3}{1 + i}$	
3	$\frac{1 - i}{1 + i}$	

4	$\dfrac{4+7i}{2+5i}$	

Square roots of negative numbers

We can use $\sqrt{ab} = \sqrt{a}\sqrt{b}$ to write square roots of negative numbers in terms of i.

e.g $\sqrt{-36} = \sqrt{36}\sqrt{-1} = 6i$

$\sqrt{-100} = \sqrt{100}\sqrt{-1} = 10i$

Write the following in terms of i as simply as possible

1. $\sqrt{-49}$

2. $\sqrt{-81}$

3. $\sqrt{-1.21}$

4. $\sqrt{-0.04}$

5. $\sqrt{-8}$

Solving quadratic equations

Complex numbers can be used to solve quadratic equations when the number being square rooted in the quadratic formula is negative.

Solve $x^2 - 2x + 2 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2\sqrt{-1}}{2}$$

$$x = 1 \ \pm i$$

Use the quadratic formula to solve the following quadratic equations.

1	$x^2 + 6x + 13 = 0$	
2	$x^2 - 4x + 13 = 0$	
3	$2x^2 - 2x + 5 = 0$	
4	$x^2 - 10x + 34 = 0$	